Introducing Mathematical Modeling to High School Students through Population Dynamics

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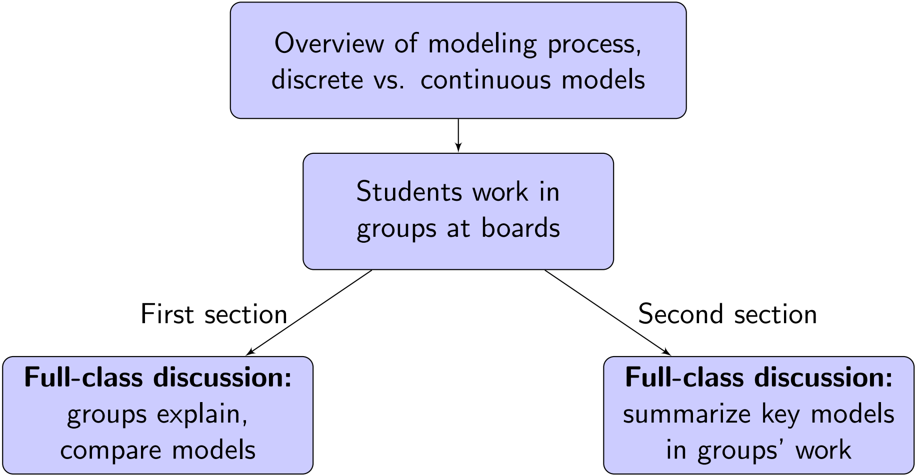
Course Context

- 50 minute class
- Developed after requests for a more advanced introduction to mathematical modeling
- Two sections: 11 students, 10 students
- Prerequisite: Comfort with thinking of derivatives as rates of change

Class Structure



Group Problems

Problem topic	**Model types**	**Source**
Newts and crayfish	predator-prey, age structure	Milligan et al. (2017)
Galician and Castilian	competition	Mira, Paredes (2005)
Pacific trophic cascade	predator-prey, competition	Estes et al. (2016)
Rabbits and woodchucks	competition	

Problem Structure

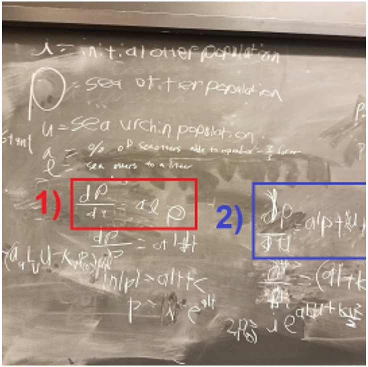
- Context
- Basic models (exponential and/or logistic), at most two species, minimal coupling
- Add complexity (coupling, more species)
- Equilibrium and other dynamics
- Control

Example: Excerpt of Trophic Cascade Problem

Sea Otter and Sea Urchin Models

Start by considering the sea otters and then the sea urchins. You'll write equations for how their populations change over time.

1. Write an equation for how the population of sea otters would change if they had infinite resources (space, food, etc.). What happens to the populations in time?
2. Now imagine that the sea urchins are the main source of food for the sea otters. Write an equation for the sea otters that is dependent on the sea urchin population.



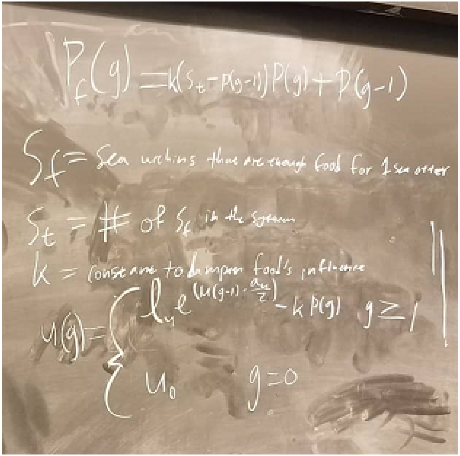


Figure: Algebraic approach to sea otter problem.

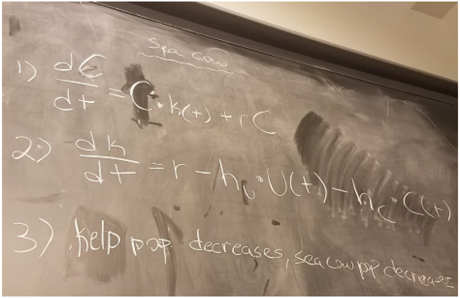


Figure: Differential equation model for sea cows, sea urchins, and kelp with physical interpretation.

What Went Well

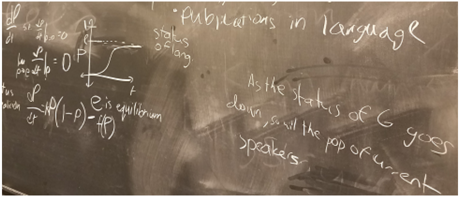


Figure: Logistic model and interpretation in context for Galician problem.

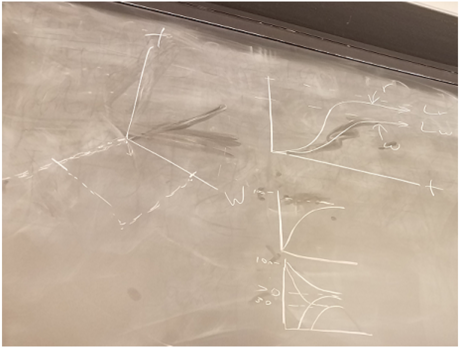


Figure: Trying different visual representations for rabbit & woodchuck problem.

Student Survey Responses

Favorite Parts:

- **Hands-on aspect (8/19):** “how hands-on it was,” “trying out something new in math,” “very interactive.“
- **Topic (7/19):** “the subject of modeling in general,” “pretty cool math,” “using differential equations and exponentials was fun,” “interesting problems.”
- **Group work (5/19):** “the interactiveness and groupwork ... helped bolster the learning process,” “working on the problems as a group.”
- **Real world (3/19):** “working in groups to solve a realistic problem,” “working on a real world problem and knowing how it could be applied.”

What Didn't Go Well

- Groups ran out of time for key questions of problems
- Too much focus on classic models instead of process
- Students with calc background wanted to solve differential equations
- Students without calc background weren't comfortable with difference/differential equations

Student Survey Responses

Recommendations for Changes:

- **Provide more background (13/19):** "more introduction," "do an example," "more background on modeling populations."
- **Leveling/grouping based on experience (2/19)**
- **Shift scaffolding (1/19):** "Try to decrease the amount of assumptions you give us.... The fun of math modeling is assumptions, and that got lost in making it mathy."

Future Changes

- Start with full-class discussion of exponential and logistic models
- Introduce equilibria and slope fields
- Maybe introduce predator-prey, competitive Lotka-Volterra
- Restructure problems to give students more modeling freedom