Newts and Crayfish

California newt populations have drastically declined because of an invasive species predator, the red swamp crayfish. These crayfish are resistant to the toxins in newts and newt eggs. The crayfish attack adult newts and eat newt eggs and larvae. Many have proposed crayfish trapping as a potential solution to the problem.

Newt Model Start by considering just the newts. Each spring, adult female newts lay eggs, which hatch into larvae in the summer and then become juveniles in the fall. A newt is a juvenile until it is 6 years old, when it becomes an adult. Adult newts live until they're around 24 years old.

Write equations for the number of adults, juveniles, and eggs each spring based on the numbers from the previous spring. You don't need to use any particular numbers; just define the variables and constants you use.

1. Are the number of eggs this spring related to last year's number of adults, juveniles, or both? Related how? How do you write that in your equations?
2. What are non-crayfish causes of death for the newts, and which age groups do those causes affect? How do you include those effects in your equations?
3. How do you represent larvae becoming juveniles and juveniles becoming adults?

Adding in Crayfish By attacking (but not necessarily killing) adult newts, crayfish disrupt mating and egg-laying. Crayfish also eat newt egg/larvae. Write new newt equations including these two effects.

1. Which population this year do the attacks of last year affect? How does this depend on the number of crayfish last year?
2. Which population this year does the predation on eggs/larvae last year affect? How does this depend on the number of crayfish last year?

Crayfish Equation We also need an equation for the crayfish! (We won't split them into age groups.)

1. How does this year's crayfish population depend on last year's?
2. Does this year's crayfish population depend on the newt populations? Why or why not? (Try to think of reasons both ways and how you might write the equation in both cases.) If so, which parts of the population?

Equilibrium We say a system is in equilibrium when it isn't changing. Here, that would mean that the populations of crayfish and newts are the same from one year to the next.

1. How would you find the equilibrium of one of your populations?
2. Go back to your no-crayfish equations and find the equilibria for the adult and juvenile newt populations.
3. How does this change with crayfish added in?
4. Find any equilibria of the crayfish population.

Crayfish Trapping

1. What are some different ways that crayfish trapping could work? How would those different ways each change the crayfish population equation?

2. Do you think it would be necessary to trap all the crayfish to save the newt population? How would you find out?

3. When would be the best times of year to trap crayfish in order to help the newt population recover?

Galician and Spanish

Galician and Castilian Spanish are two languages spoken in Spain. Castilian Spanish is the official language/dialect of the whole country and is the first or second language of 99% of the population. Galician is a co-official language in northwest Spain. Within that region, 50% of people speak Galician more often than Castilian Spanish, and another 22% spoke it sometimes. With lots of minority languages shrinking dramatically or dying out, the coexistence of the two languages is interesting.

No Bilinguals: Setting Up the Model Start by assuming that no one is bilingual. (We'll fix this bad assumption later.) Everyone we consider speaks either only Castilian Spanish or only Galician.

1. Define a variable for the percentage of the population speaking Galician and a variable for the percentage of the population speaking Spanish. Why is it useful to consider the percentage of the population instead of the number of people?

2. What are the ways in which the Galician-speaking percentage of the population can change in time? Try to be as broad as possible. What will this look like in your equation for the Galician-speaking population?

No Bilinguals: Model Details In writing equations below, you don't need to use any particular numbers. Just define the variables and constants you use.

1. What factors might affect how percentages of the population move among the two languages?

2. Try to group these factors as being related to population (number of speakers) or status (where/how the language is used and how it is seen). Do any of the factors you came up with not fit into one of these categories?

3. Think of Galician's status relative to Castilian Spanish as being a number between 0 and 1, where 0 means Galician has no status and 1 means Galician has a lot of status (and Castilian Spanish has none). Write a function for the probability of the population switching from Galician to Castilian Spanish with percentage Galician speakers and Galician's status as inputs. (Feel free to leave some part of the function, like an exponent or other constant, unknown.)

4. Given that function, what should the function for the probability of switching from Castilian Spanish to Galician look like?

5. Rewrite your Galician-speaking population equation using these functions.

No Bilinguals: Equilibrium & Stability We say a system is in equilibrium when it isn't changing. Here, that would mean that the percentage of the population speaking Galician would stay the same over time.

1. How would you find equilibria of the system from your equation?

2. Find the equilibria. Explain what's going on in the real world in each one. How does this depend on the value of Galician's status? On anything you may have left unknown in your function?

3. Imagine that the population started somewhere between two of the equilibrium points. As time went on, which equilibrium would the population move towards?

Bilingual Model Now consider three populations: Galician-only speakers, Castilian Spanish-only speakers, and bilinguals.

1. What is the relationship among these population percentages? How many equations do you need? What probability functions do you need?
2. Write new versions of the probability functions. What populations should each one depend upon? You can add another input to some or all of the functions if you need/want to. Is being bilingual of the same difficulty regardless of what the two languages are, or does it depend on the languages?
3. Find equilibria of your new equations. (Remember that all populations need to be constant in time to be at equilibrium.) How do the equilibria depend on the various inputs and parameters? What do the equilibria mean in the real world? Are there any new possibilities in this model compared to your last one?
4. Based on both of your models, what factors would you guess have helped Galician survive?

Rabbits and Woodchucks

Rabbits and woodchucks eat similar food, so when they live in the same area, they compete for food resources.

No Interaction Start by considering a situation in which the rabbits and woodchucks aren't in competition. Each population grows or declines independently of the other. You'll write equations for each population. You don't need to use any particular numbers; just define the variables and constants you use.

1. Write equations for how each population would change in time if there were infinite resources. What happens to the populations in time? How does this depend on any constants you used?

2. Now imagine there's limited food/space for each population. How might you account for this in your equations?

3. Now what happens to the populations in time? Try drawing a graph of population vs time for one of the species, starting at several different populations.

Competition Now we'll consider the two species in the same environment.

1. Which part of your equations would the presence of the other species affect? How would you include the impact of the other species? Do woodchucks have the same impact on rabbits as rabbits have on woodchucks?

Dynamics We say a system is in equilibrium when it isn't changing. Here, that would mean that the populations of rabbits and woodchucks are both constant.

1. Under what conditions is the number of rabbits constant? What about the number of woodchucks?

2. What are the equilibria of the system? What do these mean in the real world?

3. Draw a plane with woodchuck population on one axis and rabbit population on the other. In what regions of the plane is the woodchuck population growing? Where are the woodchucks mostly competing with each other? Where are they mostly competing with rabbits?

4. Draw a similar image for the rabbit population.

5. Put those two pictures together. Where are both populations growing? Both declining? One growing and one declining? Make any assumptions about values of your constants that you need to.

6. Over that picture, draw a slope field – small arrows at regular points indicating in what direction the populations are changing. (If both were increasing by equal amounts, then you would draw an arrow up and to the right at 45 degrees.)

7. Draw in some trajectories following the arrows. If a population started at a certain number of woodchucks and rabbits, what would happen to it as time went on?

Sea Otters, Sea Urchins, Kelp, and Sea Cows

In the North Pacific Ocean in the 1700s, sea otters were heavily hunted. This led to dramatic declines not only in the sea otter population but in kelp and sea cow populations as well. In fact, the Steller's sea cow went extinct.

Sea Otter and Sea Urchin Models

Start by considering the sea otters and then the sea urchins. You'll write equations for how their populations change over time.

1. Write an equation for how the population of sea otters would change if they had infinite resources (space, food, etc.). What happens to the populations in time? How does this depend on any constants you used in your equation?

2. Now imagine that the sea urchins are the main source of food for the sea otters. Write an equation for the sea otters that is dependent on the sea urchin population. (Don't write an equation for the sea urchin population yet.)

3. What happens to the sea otters when there are lots of sea urchins? When there are few?

4. Now write an equation for the sea urchins if they are hunted by the sea otters but have infinite resources. What happens to the sea otter and sea urchin populations in time?

Sea Cow Model

Now we consider the sea cow. Unlike the sea urchin, which eats kelp but lots of other food as well, the sea cow relies mainly on kelp.

You have two choices here! You can either write equations for the sea cows and the kelp, or you can exclude the kelp and just think about how sea urchins in the system affect sea cows. For now, go with either Choice 1 or Choice 2. If you have time, you can come back and try out the other one.

Choice 1:

1. Write an equation for how the sea cow population changes in time using the current kelp population. What equation that you wrote before does this look like?

2. Write an equation for the kelp population. Remember that kelp is eaten by both sea urchins and sea cows, though maybe at different rates.

3. If the sea urchin population increases, what happens to the kelp population? What does that do the sea cows?

4. If the sea cow population increases instead, does anything happen to the sea urchins? Why is this the case?

Choice 2:

1. Write equations for how the sea cow population would change in time if it had infinite resources. This should look like some equations you've written before.

2. Now imagine the sea cows have limited food. How might you account for this in your equation?

3. The sea cows compete with sea urchins for food. How might you include this in the equation?

4. If the sea urchin population increases, what happens to the sea cow population? If the sea cow population increases, what happens to the sea urchin population? Why?

Equilibrium

We say a system is in equilibrium when it isn't changing. Here, that would mean that all the populations (sea otters, urchins, sea cows, and kelp if you have it) would stay the same over time.

1. How would you find the equilibria of the system from your equation?

2. Find the equilibria. Try to explain what's going on in the real world in each one. How does this depend on the value of the various constants you used?

3. Even if the populations aren't at equilibrium, there are ways that all the species could survive. What might the populations over time look like in that case? (Think back to the last question of the sea otter and sea urchin section.)

Sea Otter Hunting

Humans began aggressively hunting sea otters. This caused a large change in the sea otter population, so we need to include it in the model.

1. What are some different ways that hunting of sea otters could be expressed in your equation? What do those different ways mean in the real world?

2. Choose one way of including sea otter hunting in your equations. How large does the hunting need to be (in terms of other constants and terms in your model) for the sea otter population to decline?

3. In your model, if the sea otter population declined, would the sea otters go extinct, or would it rebound at some point?

4. If the sea otter population goes through a very large decline, what happens to the other populations?

5. If you were using a computer to study these equations, what would you investigate? (What questions would you answer, what parameters/constants would you change, etc.)